



## University of Kerala

Discipline	Mathematics				
Course Code	UK2DSCMAT101				
Course Title	Integration and Multivariate Calculus				
Type of Course	DSC				
Semester	II				
Academic Level	100-199				
Course Details	Credit	Lecture per week	Tutorial per week	Practical per week	Total Hours per week
	4	3	-	2	5
Pre-requisites	1. Derivative of functions    2. Vectors				
Course Summary	This course equip the students to find the integral of functions, its applications, partial derivatives of functions and to know about the basic concepts of vector valued functions				

## Detailed Syllabus

Module	Unit	Contents	Hrs
<b>I</b>		<b>Integration</b>	<b>9</b>
	1	The Indefinite Integral: Antiderivatives, The Indefinite Integrals, Integration formulas, Properties of the indefinite integral, Integral curves.	
	2	Integration by substitution (excluding Integration using computer algebra systems)	
	3	Evaluation of definite integral by substitution	
	4	Integrals of logarithmic Functions	
	5	Integrals of exponential functions	
		Chapter 4: Section 4.2, 4.3, 4.9 Chapter 6: section 6.2 (integration only), 6.3 (integration only) of Text[1]	

Module	Unit	Contents	Hrs
<b>II</b>	<b>Applications of Integration</b>		<b>9</b>
	6	Area between two curves	
	7	Volume by Slicing: (excluding other axis of revolution)	
	8	Length of a plane curve(excluding finding arc length by numerical methods)	
	Chapter 5: Section 5.1, 5.2, 5.4 of Text[1]		
<b>III</b>	<b>Vector Calculus 1</b>		<b>9</b>
	9	Introduction to vector valued functions: Parametric curves in 3-space, vector-valued functions, vector form of a line segment	
	10	Calculus of vector valued functions (excluding antiderivatives of vector-valued functions)	
	11	Unit tangent and normal vectors(excluding binormal vectors in 3-space)	
	Chapter 12: Section 12.1, 12.2, 12.4 of Text[1]		
<b>IV</b>	<b>Partial Differentiation</b>		<b>9</b>
	12	Functions of two or more variables(Notation and terminology only)	
	13	Partial derivatives(excluding estimating partial derivatives from tabular data, partial derivatives and continuity, equality of mixed partials, wave equations)	
	14	The Chain rule	
	15	Maxima and minima of functions of two variables	
	Chapter 13: Section 13.1, 13.3, 13.5, 13.8 of Text[1]		
<b>V</b>	<b>Teacher designed module - suggested topics</b>		<b>9</b>
	For internal assessment examinations only.		
	16	An overview of area problem	
	17	Volume by other axis of revolution	
	18	Area of a surface of revolution	
	19	Curvature	
	20	Equality of mixed partials, wave equations	
	21	Langrange mutipliers	
	Sections from Text [1]		

## Topics for Practical sessions – 30 hours

1. Introducing the SAGEMATH interface, SAGE cell server; basic arithmetic involving operators  $+$ ,  $-$ ,  $/$ , exponentiation; functions like  $\sin$ ,  $\cos$ ,  $\tan$ ,  $e$ ,  $\log$ ,  $\text{sqrt}$ , constant  $\pi$   
Ref: P1, or section 2.3 of P2
2. Defining and using lists, dictionaries, sets, and accessing elements in lists and dictionaries  
Ref: section 5.1, 5.3, 5.4 of P3

3. Defining variables using `var`, defining polynomials, polynomial functions, evaluating them Ref: P3 or section 1.4 of P4
4. Using `integrate` command to compute indefinite and definite integrals  
Ref : Section 3.3.4 of P2
5. Sketching graphs of curves using `plot`  
Ref : Section 6.1 of P2
6. Defining curves, finding area between two curves  
Ref : Section 6.1 of P4
7. Finding volumes of solids of revolution, finding arc length  
Ref : Section 6.3 of P4
8. Defining parametric functions, sketching the graphs  
Ref: P5, Section 6.1 of P2
9. `diff` command to find derivatives of standard functions, polynomials  
Ref: Section 3.1 of P4
10. Finding derivatives of vector valued functions
11. Defining vectors, finding their dot and cross products, finding norm of vectors  
Ref: Section 3.3.5 of P2
12. Computing unit tangent and normal vectors, sketching the curve and plotting these vectors  
Ref : P6
13. Defining functions of multiple variables, evaluating them at certain points, differentiating them
14. Solving polynomial equations and equations involving standard functions  
Ref : Section 2.2 of P7
15. Computing maxima and minima of multivariable functions  
Ref : Section 4.3 of P4
16. Computing maxima and minima using Lagrange multiplier technique  
Ref : Section 4.18 of P8
17. Plotting in 3-dimension, marking optimal points on the plots obtained through the maxima minima problems  
Ref : P9, Section 7.1 of P2

### **Problems for the practical examination**

1. Computing indefinite and definite integrals of standard functions (trigonometric, log, e, polynomials)
2. Demonstrate the plot command with various options (line style, color, thickness etc)
3. Finding area between two curves, sketching them

4. Finding volumes of solids of revolution, sketching the curves and solids
5. Defining multivariable functions, evaluating them, differentiation them
6. Defining and solving polynomial equations, evaluating them
7. Defining vectors, finding their dot, cross products, norm
8. Computing unit tangent vectors plotting them on the vector curves
9. computing maxima and minima directly (w/o Lagrange multiplier)
10. computing maxima and minima directly using Lagrange multiplier

A record should be maintained with at least 7 problems from the above. Each problem in the record must have a description of the problem, algorithm (step by step procedure), commands used, input given and output obtained accordingly. For the ESE, from the list of above 10 problems, the student should be able to answer two selected (from the 7 available in the record) by the examiner.

## Textbooks

1. Howard Anton, Irel Bivens, Stephens Davis, *Calculus* 10<sup>th</sup> Edition Wiley, 2012.

## References

1. Erwin Kreyszig, *Advanced Engineering Mathematics*, 10<sup>th</sup> Edition Wiley, 2018.
2. Ian Sneddon, *Elements of Partial Differential Equations*, Mc Graw- Hill, 2013.
3. Peter. V. O Neil, *Advanced Engineering Mathematics*, Thompson Publications, 2007.
4. M. D. Raisinghaniya, *Ordinary and Partial Differential Equations*, S Chand 18<sup>th</sup> Edition, 2008.
5. G. F. Simmons, *Differential Equations with Applications and Historical Notes*, Tata McGraw-Hill, 2003.
6. G. B. Thomas, R. L. Finey, *Calculus*, 9<sup>th</sup> Edition, Addison-Weseley Publishing Company, 2004.

## Resources for practical sessions

- P1. Sagemath documentation – Introductory Sage Tutorial <https://doc.sagemath.org/html/en/prep/Intro-Tutorial.html>
- P2. Saskia Roos, Michael Jung, *An Introductory Course on Sage, Lecture Notes* [https://www.math.uni-potsdam.de/fileadmin/user\\_upload/An\\_Introductory\\_Course\\_on\\_Sage.pdf](https://www.math.uni-potsdam.de/fileadmin/user_upload/An_Introductory_Course_on_Sage.pdf)

- P3. Sagemath documentation – Symbolic variables <https://doc.sagemath.org/html/en/reference/calculus/sage/calculus/var.html>
- P4. Tuan A. Le, Hieu D. Nguyen, SageMath Advice for calculus <https://users.rowan.edu/~nguyen/sage/SageMathAdviceforCalculus.pdf>
- P5. Sagemath documentation – Sage Quickstart for Multivariable Calculus <https://doc.sagemath.org/html/en/prepare/Quickstarts/Multivariable-Calculus.html>
- P6. Sagemath documentation – Parametric plots [https://doc.sagemath.org/html/en/reference/plot3d/sage/plot/plot3d/parametric\\_plot3d.html#sage.plot.plot3d.parametric\\_plot3d.parametric\\_plot3d](https://doc.sagemath.org/html/en/reference/plot3d/sage/plot/plot3d/parametric_plot3d.html#sage.plot.plot3d.parametric_plot3d.parametric_plot3d)
- P7. P. Zimmermann *et al*, Computational Mathematics with SageMath, <https://www.sagemath.org/sagebook/english.html>
- P8. Gregory V. Bard, Sage for Undergraduates <http://www.people.vcu.edu/~clarson/bard-sage-for-undergraduates-2014.pdf>
- P9. SageMath documentation – 3D Graphics <https://doc.sagemath.org/html/en/reference/plot3d/index.html>

## Course Outcomes

CO No.	Upon completion of the course the graduate will be able to	PO/PSO	Cognitive Level	Knowledge Category	Lecture(L) Tutorial (T)	Practical (P)
CO 1	Understand the fundamental concepts of Integration and Vector valued function.	PSO1, 2 PO1, 3, 6, 7, 8	U, Ap	F,C	L	
CO 2	Analyze the various techniques both in Integration and in Vector Calculus	PSO 2,3 PO1, 2, 3, 6, 7, 8	U, An	C,P	L	
CO 3	Develop problem-solving techniques	PSO 1,2,3,4, PO1, 2, 3, 6, 7, 8	An, E	P	L	

(R-Remember, U-Understand, Ap-Apply, An-Analyse, E-Evaluate, C-Create)  
(F-Factual, C-Conceptual, P-Procedural, M-Metacognitive)

## Mapping of CO with PSOs and POs

	PSO1	PSO2	PSO3	PSO4	PSO5	PSO6	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8
CO1	3	1	2	2	3	3	3	-	1	-	-	2	1	1
CO2	1	3	1	2	1	1	1	1	3	-	-	2	1	1
CO3	1	3	2	3	2	3	1	3	3	-	-	3	1	1

(- -Nil, 1-Slightly/Low, 2-Moderate/Medium, 3-Substantial/High)

## Assessment Rubrics

- Quiz/Assignment/Discussion/Seminar
- Midterm Exam
- Programming Assignments
- End Semester Exam

## Mapping of COs to Assessment Rubrics

	Internal Examination	Assignment	Project Evaluation	End Semester Exam
CO1	✓	—	—	✓
CO2	✓	✓		✓
CO3	✓	✓		✓