University of Kerala

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|---|---|---|---|---|---|
| Discipline | Mathematics | | | | |
| Course Code | UK3DSCMAT205 | | | | |
| Course Title | Multivariate Calculus and Multiple Integrals | | | | |
| Type of Course | DSC | | | | |
| Semester | III | | | | |
| Academic Level | 200-299 | | | | |
| Course Details | Credit | Lecture per week | Tutorial per week | Practical per week | Total Hours per week |
| | 4 | 3 | - | 2 | 5 |
| Pre-requisites | 1. Differentiation 2. Integration | | | | |
| Course Summary | This course gives an insight into Multi Variable Calculus, Multiple Integrals, Vector Calculus | | | | |

# Detailed Syllabus

| Module | Unit | Contents | Hrs |
|---|---|---|---|
| **I** | | **Multivariable Calculus** | **9** |
| | 1 | Functions of Two or More Variables, Limits and Continuity (Chapter13: Sections 13.1, 13.2 of Text[1], Continuity at Boundary Points and Extensions of Three Variables may be omitted.) | |
| | 2 | Partial Derivatives, The Chain Rule (Chapter 13 Section 13.3, 13.5 of Text[1]) (Estimating Partial Derivatives From Tabular Data, Partial Derivatives and Continuity, The Wave Equation may be omitted) | |
| **II** | | **Multiple Integrals** | **9** |
| | 3 | Double Integrals, Double Integrals over Non-rectangular Regions (Chapter 14: Section 14.1, 14.2 of Text[1] | |

| Module | Unit | Contents | Hrs |
|---|---|---|---|
| | 4 | Double Integrals in Polar Coordinates (Chapter 14: Section 14.3 of Text[1]) | |
| | 5 | Triple Integrals (Chapter 14: Section 14.5 of Text 1). Definition of Triple Integral, Properties of Triple Integrals, Evaluating Triple Integrals over Rectangular Boxes. | |
| **III** | **Vector Differentiation** | | **9** |
| | 6 | Vector fields (Definition), inverse square fields, Gradient fields, Conservative Fields and potential functions, Divergence and Curl, the $\nabla$ operator (Chapter 15: Section 15.1 of Text[1]) | |
| **IV** | **Vector Integration** | | **9** |
| | 7 | Line integrals, Integrating a vector field along a curve - Exercise Set 15.2- problems 15-30, 33-36, 41-46. (Chapter 15: Section 15.2 of Text[1]) | |
| | 8 | Independence of Path; Conservative Vector Fields, Green's Theorem (Chapter 15: Sections 15.3, 15.4 of Text[1]) | |
| **V** | **Suggestion for Teacher Designed Module** | | **9** |
| | 9 | Level Curves, Contour Plots Using Technology, Level Surfaces (Chapter 13 of Module 1), Directional Derivatives, Maxima and Minima Of Functions of Two Variables(Chapter 13: 13.6, 13.7 of Text 1) | |
| | 10 | Triple Integrals(Chapter 14: Section 14.5 of Text 1), Evaluating Triple Integrals over more General regions, Volume Calculated as a triple Integral, Integration in other orders. | |

# Practical sessions and examinations – 30 hours

All the topics mentioned above can be used for practical sessions using SageMath software. Some useful resources for solving these problems using the SageMath software are given against each problem/type of problem.

1. Define a function with two/three variables and evaluate it at any given point.
2. Sketch the graphs and level surfaces of two variable functions and (using 3 dimensional plot)
3. Evaluate the limit of two variable functions
4. Compute partial derivatives of two/three variable functoins and evaluate them at given points.
5. Evaluate double integrals in rectangular co-ordinates.
6. Convert representation of points among rectangular, polar, spherical co-ordinate systems
7. Compute the maxima and minima of two variable functions using second partials test

8. Sketch vector fields.
9. Compute the gradient of function.
10. Compute the divergence, curl.
11. Verify Green's theorem for selected functions.

## Resources for practical sessions

1. SageMath – documentation
   https://doc.sagemath.org/html/en/tutorial/introduction.html
2. Online SageMath server https://sagecell.sagemath.org/
3. Graphing Functions of two variables using Technology
   https://doc.sagemath.org/html/en/tutorial/tour_algebra.html
4. Maxima and Minima Of Functions Of Two Variables
   https://wiki.sagemath.org/interact/calculus#Root_Finding_Using_Bisection
5. Sagemath documentation – Sage Quickstart for Multivariable Calculus
   https://doc.sagemath.org/html/en/prep/Quickstarts/Multivariable-Calculus.html
6. Tutorial: Vector Calculus in Euclidean Spaces https://doc.sagemath.org/html/en/thematic_tutorials/vector_calculus.html
7. Sagemath documentation – Parametric plots
   https://doc.sagemath.org/html/en/reference/plot3d/sage/plot/plot3d/parametric_plot3d.html#sage.plot.plot3d.parametric_plot3d.parametric_plot3d

A record should be maintained with atleast 7 problems from the main topics/teacher designed topics. Each problem in the record must have a description of the problem, algorithm (step by step procedure), commands used, input given and output obtained accordingly. For the ESE, from the list of above 10 problems, the student should be able to answer two selected (from the 7 available in the record) by the examiner.

## Textbook

1. H Anton, I Bivens, S Davis, Calculus, 10th Edition, John Wiley & Sons, 2012.

# References


1. Joel Hass, Maurice D. Weir, Thomas' Calculus, Early Transcendentals, 12th Edition, Addison-Weseley Publishing Company, 2004.

2. Joel Hass, Maurice D. Weir, Thomas' Calculus Early Transcendentals, 12th Edition, Addison-Weseley Publishing Company, 2004.

3. G B Thomas, R L Finney, Calculus, 9 th Edition, Addison-Weseley Publishing Company, 2004.

# Course Outcomes

| CO No. | Upon completion of the course the graduate will be able to | PO/PSO | Cognitive Level | Knowledge Category | Lecture(L) Tutorial (T) | Practical (P) |
|---|---|---|---|---|---|---|
| CO 1 | Define the concept of functions of several variables, their limit, continuity and derivative | PSO 1, PO1, 2, 6 | U | F,C | L | |
| CO 2 | Illustrate various applications of multivariable calculus | PSO 2, 4, PO1, 2, 6 | Ap, An | P | L | |
| CO 3 | Describe the concepts Vector fields, Gradient fields, potential functions and vector integrals | PSO 1, PO1, 2, 6 | U | F,C | L | |
| CO 4 | Apply vector integrals to find areas | PSO 3, 4, PO1, 2, 6 | Ap, An | P | L | |

**(R-Remember, U-Understand, Ap-Apply, An-Analyse, E-Evaluate, C-Create)**
**(F-Factual, C-Conceptual, P-Procedural, M-Metacognitive)**

# Mapping of CO with PSOs and POs

| | PSO1 | PSO2 | PSO3 | PSO4 | PSO5 | PSO6 | PO1 | PO2 | PO3 | PO4 | PO5 | PO6 | PO7 | PO8 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| CO1 | 3 | - | - | - | - | - | 2 | 1 | - | - | - | 3 | - | - |
| CO2 | - | - | 3 | 3 | - | - | 2 | 1 | - | - | - | 3 | - | - |
| CO3 | 3 | - | - | - | - | - | 1 | 1 | - | - | - | 2 | - | - |
| CO4 | - | - | 3 | 3 | - | - | 2 | 1 | - | - | - | 3 | - | - |

**( - -Nill, 1-Slightly/Low, 2-Moderate/Medium, 3-Substantial/High)**

# Assessment Rubrics

- Quiz/Assignment/Discussion/Seminar

- Midterm Exam
- Programming Assignments
- Final Exam

## Mapping of COs to Assessment Rubrics

| | Internal Examination | Assignment | Project Evaluation | End Semester Exam |
|---|---|---|---|---|
| CO1 | ✓ | | | ✓ |
| CO2 | ✓ | ✓ | | ✓ |
| CO3 | ✓ | | | ✓ |
| CO4 | ✓ | ✓ | | ✓ |