



## University of Kerala

|                |  |                     |                      |           |                         |
|----------------|--|---------------------|----------------------|-----------|-------------------------|
| Discipline     | Mathematics  |                     |                      |           |                         |
| Course Code    | UK3DSCMAT212   |                     |                      |           |                         |
| Course Title   | Vector Calculus  |                     |                      |           |                         |
| Type of Course | DSC  |                     |                      |           |                         |
| Semester       | III  |                     |                      |           |                         |
| Academic Level | 200-299  |                     |                      |           |                         |
| Course Details | Credit   | Lecture<br>per week | Tutorial<br>per week | Practical | Total<br>Hours per week |
|                | 4  | 3                   |                      | 2         | 5                       |
| Pre-requisites | 1. Awareness on polynomials<br>2. Knowledge on the concepts of functions, differentiation and basic geometry |                     |                      |           |                         |
| Course Summary | This course includes theory of equations, differential calculus, polar co-ordinates and conic sections       |                     |                      |           |                         |

### Detailed Syllabus

| Module                                      | Unit | Contents  | Hrs      |
|---|------|---|----------|
| <b>I</b>                                    |      | <b>Vector Valued Functions</b>  | <b>9</b> |
|   | 1    | Introduction to vector valued Functions, Parametric Curves in 3-Space - The parametric equations (introduction only) vector valued functions (introduction only) vector form of a line segment (introduction only)  |          |
|   | 2    | Calculus of vector-valued Functions - Limits and Continuity, Geometric interpretations of limits, Derivatives, Geometric interpretation of the derivative, derivative rules Derivatives of dot and cross products (fundamentals only) Integrals of vector valued functions and integral rules (fundamentals only) |          |
| Chapter 12: Sections 12.1, 12.2 of Text [1] |      |   |          |



| Module     | Unit  | Contents  | Hrs      |
|------------|---|---|----------|
| <b>II</b>  | <b>Tangents and Curvature</b>   |   | <b>9</b> |
|            | 3   | Unit Tangent, Normal and Binormal vectors (introduction only) Normal and Tangential Components of Acceleration Curvature, radius of curvature,  |          |
|            | Chapter 12: Section 12.4, 12.5 of Text [1]  |   |          |
| <b>III</b> | <b>Vector Differentiation</b>   |   | <b>9</b> |
|            | 4   | Vector fields (Definition), inverse square fields, Gradient fields, Conservative Fields and potential functions, Divergence and Curl, the $\nabla$ operator   |          |
|            | Chapter 15: Section 15.1 of Text[1]   |   |          |
| <b>IV</b>  | <b>Vector Integration</b>   |   | <b>9</b> |
|            | 5   | Line integrals, Integrating a vector field along a curve - Exercise Set 15.2- problems 15-30, 33-36, 41-46, Independence of Path; Conservative Vector Fields  |          |
|            | 6   | Chapter 15: Section 15.2, 15.3 of Text[1]   |          |
| <b>V</b>   | <b>Suggestions for teacher designed module</b>  |   | <b>9</b> |
|            | For internal assessment examinations only.  |   |          |
|            | 7   | Arc length parametrization<br>Motion along a curve<br>Green's Theorem<br>Surface integrals evaluating surface integrals, Flux, evaluation of flux integrals<br>The divergence theorem (without proof) using the divergence theorem to find flux. Stoke's theorem (all without proof)<br>Relationships between Green's theorem and Stoke's theorem |          |
|            | These topics can be found in Chapter 12 Sections 12.3, 12.6, Chapter 15 Sections 15.4 to 15.8 |   |          |

## Practical sessions and examinations – 30 hours

All the topics mentioned above can be used for practical sessions using SageMath software. Some useful resources for solving these problems using the SageMath software are given against each problem/type of problem.

1. Defining parametric curves
2. Computing limits of vector valued functions
3. Computing dot products
4. Computing cross product
5. Differentiating dot and cross products
6. Computing unit tangent vector



7. Computing normal vector
8. Computing curvature
9. Sketching vector fields
10. Computing gradient, divergence, curl

## Textbook

**Text 1** H Anton, I Bivens, S Davis, *Calculus Late Transcendentals*, 10<sup>th</sup> Edition, John Wiley & Sons.

## Resources for practical sessions

1. Sagemath documentation – Introductory Sage Tutorial <https://doc.sagemath.org/html/en/prep/Intro-Tutorial.html>
2. Saskia Roos, Michael Jung, *An Introductory Course on Sage, Lecture Notes* [https://www.math.uni-potsdam.de/fileadmin/user\\_upload/An\\_Introductory\\_Course\\_on\\_Sage.pdf](https://www.math.uni-potsdam.de/fileadmin/user_upload/An_Introductory_Course_on_Sage.pdf)
3. Sagemath documentation – Symbolic variables <https://doc.sagemath.org/html/en/reference/calculus/sage/calculus/var.html>
4. Tuan A. Le, Hieu D. Nguyen, SageMath Advice for calculus <https://users.rowan.edu/~nguyen/sage/SageMathAdviceforCalculus.pdf>
5. Sagemath documentation – Sage Quickstart for Multivariable Calculus <https://doc.sagemath.org/html/en/prep/Quickstarts/Multivariable-Calculus.html>
6. Sagemath documentation – Parametric plots [https://doc.sagemath.org/html/en/reference/plot3d/sage/plot/plot3d/parametric\\_plot3d.html#sage.plot.plot3d.parametric\\_plot3d.parametric\\_plot3d](https://doc.sagemath.org/html/en/reference/plot3d/sage/plot/plot3d/parametric_plot3d.html#sage.plot.plot3d.parametric_plot3d.parametric_plot3d)
7. P. Zimmermann *et al*, Computational Mathematics with SageMath, <https://www.sagemath.org/sagebook/english.html>
8. Gregory V. Bard, Sage for Undergraduates <http://www.people.vcu.edu/~clarson/bard-sage-for-undergraduates-2014.pdf>
9. SageMath documentation – 3D Graphics <https://doc.sagemath.org/html/en/reference/plot3d/index.html>



## References

- Ref. 1** Joel Hass, Maurice D. Weir, *Thomas' Calculus Early Transcendentals*, 12<sup>th</sup> Edition, Addison-Weseley Publishing Company, 2004.
- Ref. 2** J Stewart, *Calculus with Early Transcendental Functions*, 7<sup>th</sup> Edition, Cengage India Private Limited.
- Ref. 3** G B Thomas, R L Finney, *Calculus*, 9th Edition, Addison-Weseley Publishing Company, 2004.

## e-resources

1. <https://www.sagemath.org/help.html>

## Course Outcomes

| CO No. | Upon completion of the course the graduate will be able to   | PO/PSO                                      | Cognitive Level | Knowledge Category | Lecture(L)<br>Tutorial (T) | Practical (P) |
|--------|--|---|-----------------|--------------------|----------------------------|---------------|
| CO 1   | Understanding of Vector-Valued Functions and Parametric Curves   | PSO 1                                       | U               | F, C               | L                          |               |
| CO 2   | Ability to Parametrize Curves and Calculate Arc Length   | PSO1, PSO2, PSO3, PSO4, PO1, PO2            | R, U, E, Ap, An | P, C, F            | L                          |               |
| CO 3   | Application of Line and Surface Integrals  | PSO2, PSO3, PO1, PO2                        | U, Ap, E        | P, F, C            | L                          |               |
| CO 4   | Analyze and solve complex problems involving vector-valued functions and parametric curves in three-dimensional space. | PSO1, PSO2, PSO3, PSO4, PSO6, PO1, PO2, PO3 | U, Ap, An, E, C | P, M               | L                          |               |

(R-Remember, U-Understand, Ap-Apply, An-Analyse, E-Evaluate, C-Create)



(F-Factual, C-Conceptual, P-Procedural, M-Metacognitive)

## Mapping of CO with PSOs and POs

|     | PSO1 | PSO2 | PSO3 | PSO4 | PSO5 | PSO6 | PO1 | PO2 | PO3 | PO4 | PO5 | PO6 | PO7 | PO8 |
|-----|------|------|------|------|------|------|-----|-----|-----|-----|-----|-----|-----|-----|
| CO1 | 3    | -    | -    | -    | -    | -    | 3   | -   | -   | -   | -   | -   | -   | -   |
| CO2 | -    | 3    | 2    | 3    | -    | -    | -   | 3   | 2   | -   | -   | -   | -   | -   |
| CO3 | 3    | -    | -    | -    | -    | -    | 3   | -   | -   | -   | -   | -   | -   | -   |
| CO4 | -    | 3    | 3    | -    | -    | -    | 3   | 2   | -   | -   | -   | -   | -   | -   |

( - -Nil, 1-Slightly/Low, 2-Moderate/Medium, 3-Substantial/High)

## Assessment Rubrics

- Quiz/Assignment/Discussion/Seminar
- Midterm Exam
- Programming Assignments
- Final Exam

## Mapping of COs to Assessment Rubrics

|     | Internal Examination | Assignment | Project Evaluation | End Semester Exam |
|-----|----------------------|------------|--------------------|-------------------|
| CO1 | ✓                    |            |                    | ✓                 |
| CO2 | ✓                    | ✓          |                    | ✓                 |
| CO3 | ✓                    |            |                    | ✓                 |
| CO4 | ✓                    | ✓          |                    | ✓                 |

